Investigation of the impact of six factors on the portfolio returns

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Abstract: Several factors determine the stock returns, some of them are more influential. The purpose of this research was to find the decisive factors among the six candidate factors and the function that represents the return of each portfolio in terms of the return of different factors. This paper introduces the descriptive analysis of the monthly return of each factor, the regression model, and the analysis of the model. Through the analysis of various parameters and F-test, factors MKT, SMB, and HML were found to have the most relevance to our portfolio returns. This research can be helpful to better analyze the factors that influence the return rate of the stock portfolio in daily stock investment so that investors can make some more informed investments.

1. Introduction

In the world of finance, portfolio investment is a crucial part for investors and managers to carry out a substantial return on stocks. Since one decision in investment can impact a huge amount of individuals' and firms' wealth, it is very important to take the portfolio approach when investing in stocks. Diversification is one big factor that a portfolio can bring to investors as putting all eggs in one basket is extremely risky. The goal for the portfolio investment is to maximize profits while minimizing risks. Since the importance of the portfolio investment is not negligible, group members are curious to explore what factors could impact our stock portfolio returns. Among all models that have been established by previous experts in the financial world, we focus our research on the three-factor model and the five-factor model to discover the most influential candidate factors in the sample stock portfolio we take from Yahoo Finance.

The single index model "is a statistical model of security returns [that] specifies systematic uncertainty [and] unique uncertainty" [1]. The word "single" in the Single Index Model means that the model assumes that "only 1 macroeconomic factor that causes systemic risk [can affect] all stock returns and can be represented by the rate of return on a market index" [2]. The single index model has been widely used by financial experts across the field. Niranjan Mandal employed the single-index model to analyze the "BSE SENSEX market performance index" and tested "its application to construct an optimal portfolio" [3]. The single index model is also employed by researcher Arenas to project the "Expert Betas" that could "estimate the future beta of each financial asset" [4]. Besides the projection of the future financial asset, the single-index model is also utilized by Michael McAleer to "forecast value-at-risk thresholds" [5]. Overall, the Index Model has helped many financial experts in their research to explore factors that could affect portfolio performances.

The Three-Factor Model was developed by Eugene Fama and Kenneth French in 1992 that "expands on CAPM by adding size risk and value risk factors to the market risk factor [with the emphasis that] value and small-cap stocks outperform markets regularly" [6]. The Three-Factor Model has been tested for its validity by real-world application from the Istanbul Stock exchange by

researcher Veysel Eraslan when he gathers “monthly excess stock returns over the period from 2003 to 2010” [7]. Besides the validity test in Istanbul, the three-factor model has also been put in the test for Africa’s largest stock exchange JSE limited, and it was determined by the researcher Basiewicz that the Three-Factor Model "could be used in expected return estimation" [8]. The Five-Factor Model is an extension to the Three-Factor Model that "added two factors, profitability and investment" as the previous model was not sufficient to "explain some anomalies nor the cross-sectional variation in expected returns" [9]. The Five-Factor Model was also being tested by Japanese researchers Keiichi Kubota and Hitoshi Takehara for “the pricing structure of stocks with long-run data for Japan from 1978 to 2014”. Unfortunately, the model did not represent “the best benchmark pricing model for Japanese data” [10].

The Index Model and Factor Model have been widely utilized by researchers around the globe to test theories, market performances, and the validity of the model itself. Although there are great and bad representations of each model in different markets, it is still convincing for us to utilize in our research as its validity can be applied to the general market.

Therefore, in this research, we derived data from Yahoo Finance on monthly prices of six portfolios from the timespan July 1963 to September 2019 and employed the three-factor model and five-factor models on selected candidate factors to test out which factors have the most correlation to the portfolio return. After running regression analysis in the R program, we compare the coefficients of six factors and analyses that RMW, CMA, and MOM have little effects on our model, while MKT, SMB, and HML are more relevant to our return rate.

Our research structure layout for this research will be as follows: Introduction, Data, Method, Results, Analysis, and Conclusion. In the Data section, we will introduce our factors and briefly introduce each one. Besides the brief introduction, we will also conduct a descriptive table with statistical data and discount charts that display discount monthly return rate and cumulative return rate of 6 factors. In the Method section, we will introduce the index mode, three-factor model, and five-factor model that correspond to our six candidate factors and specify the approach that we will take to analyses the data. In the Results Analysis Section, we will analyses all the data that is obtained from the approach we take in the method section and introduce our findings. We will recap the background knowledge, our key methods that lead to our findings, the analysis, and its findings, including future improvements on the study.

2. Data

Our data, obtained through Yahoo Finance, includes monthly prices for six portfolios, rf(risk-free return), and six factors which are MKT(the excess return on the value-weight market portfolio), SMB(the excess return on a diversified portfolio of small stocks minus the excess return on a diversified portfolio of big stocks (i.e. the size effect)), HML(the excess return spread of cheap minus expensive stocks (i.e. the value effect)), RMW(the difference between the excess returns on diversified portfolios of stocks with robust and weak profitability), CMA(the excess return spread of firms that invest conservatively minus aggressively) and MOM(the excess return spread of firms with high prior return minus low prior return) from July 1963 to September 2019.

Table 1. Descriptive statistics of variables.

	MKT	SMB	HML	RMW	CMA	MOM
nbr. val	675	675	675	675	675	675
nbr. null	1	3	2	2	3	1
nbr.na	0	0	0	0	0	0
min	-23.24	-14.91	-11.18	-18.33	-6.86	-34.39
max	16.1	18.32	12.87	13.33	9.56	18.36
range	39.34	33.23	24.05	31.66	16.42	52.75
sum	358.31	153.26	209.22	176.66	186.43	442.05
median	0.88	0.08	0.23	0.24	0.13	0.74
mean	0.53083	0.227052	0.309956	0.261719	0.276193	0.654889
SE. mean	0.169075	0.115972	0.108218	0.083109	0.076778	0.161321
CI.mean.0.95	0.331978	0.227711	0.212484	0.163183	0.150753	0.316753
var	19.29587	9.078483	7.904951	4.662242	3.979031	17.56662
std. dev	4.392706	3.013052	2.811574	2.159223	1.994751	4.191255
coef. var	8.275172	13.27033	9.070895	8.250171	7.222318	6.399948

Then we used R software to draw a descriptive analysis table of the six factors with the minimum, maximum, average, median, range, standard difference, and a series of descriptive statistics of the six factors listed on it.

It can be seen from Table 1 that the mean values of these six factors are all greater than 0. MKT had the highest standard deviation (4.39) and CMA had the lowest standard deviation (1.99). The standard deviation measures the dispersion of the data set relative to the mean, with volatile stocks typically having high standard deviations and stable stocks typically having low standard deviations.

Then R software was used to draw the discount charts of monthly return rate and cumulative return rate of 6 factors.

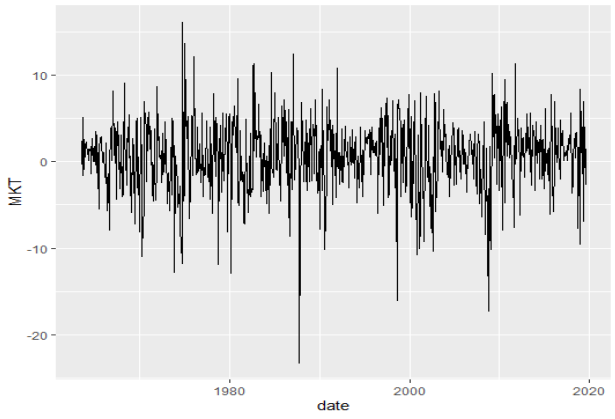

Figure 1. Monthly return rate of MKT.

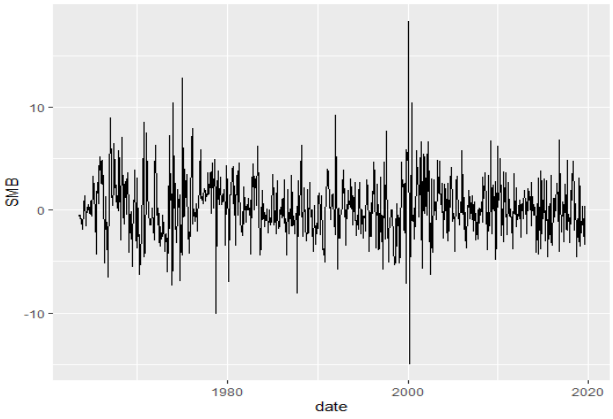

Figure 2. Monthly return rate of SMB.

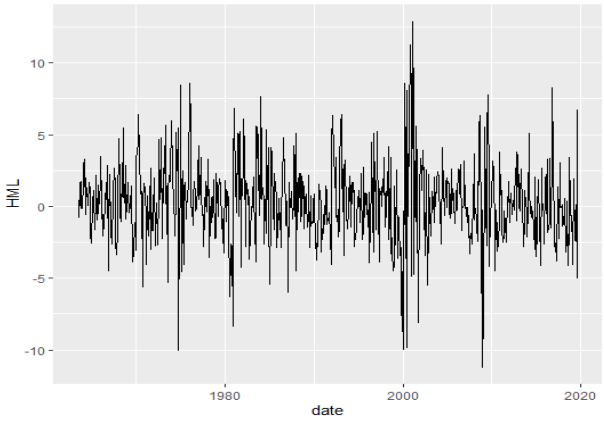

Figure 3. Monthly return rate of HML.

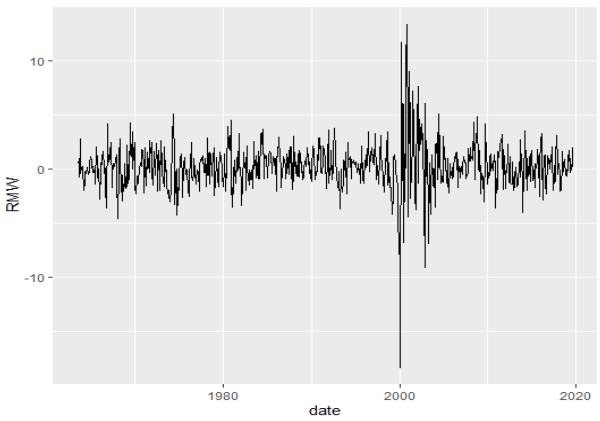

Figure 4. Monthly return rate of RMW.

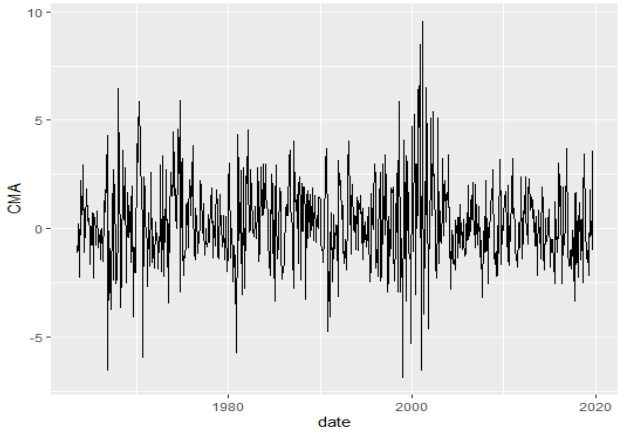

Figure 5. Monthly return rate of CMA.

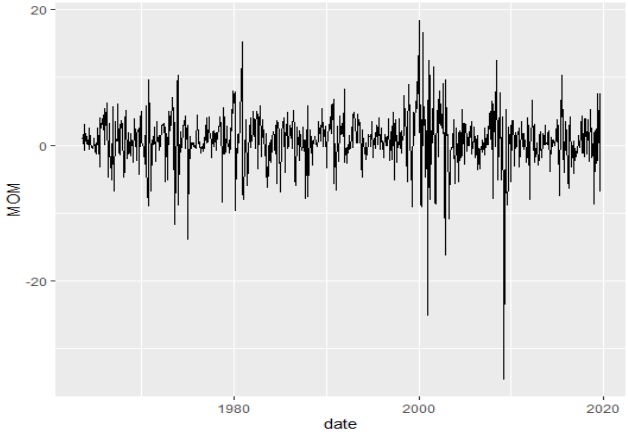

Figure 6. Monthly return rate of MOM.

As can be seen from figure 1 to figure 6, from July 1, 1963 to September 1, 2019, the monthly return rate of SMB, HML, RMW, CMA, and MOM all had a volient fluctuation around 2000. Corresponding to the standard deviation obtained above, the curves of RMW and CMA are most smooth and which of MKT and MOM are more abrupt.

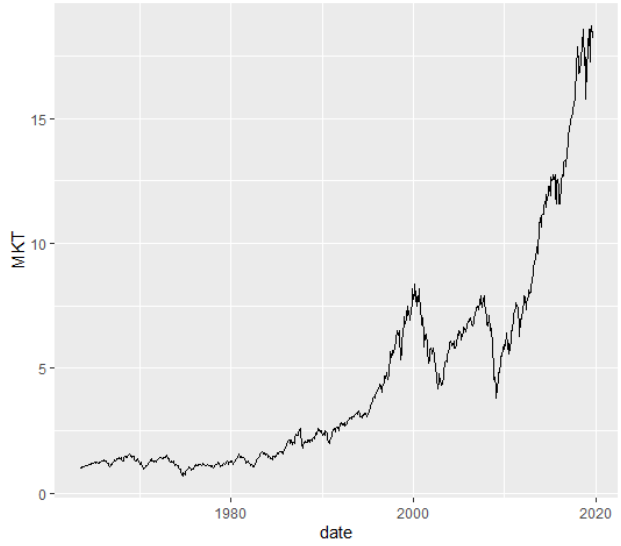

Figure 7. Cumulative return rate of MKT.

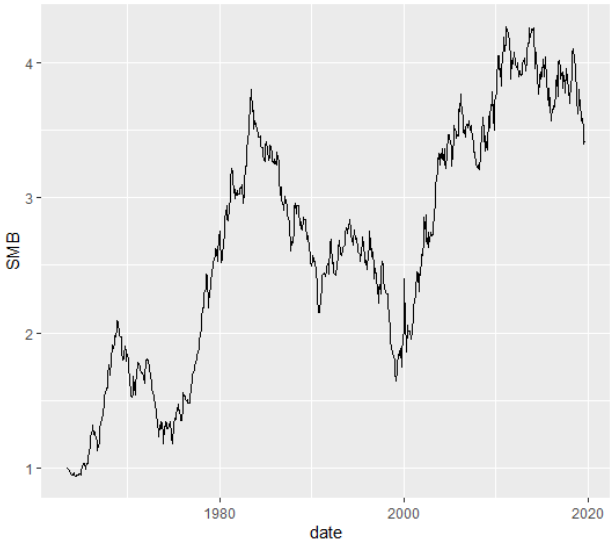

Figure 8. Cumulative return rate of SMB.

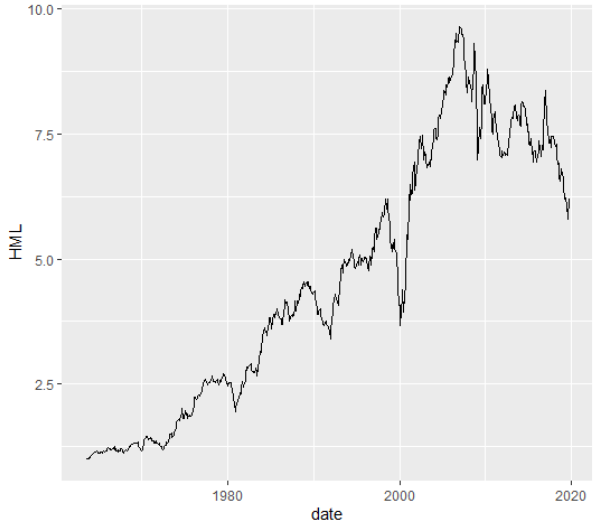

Figure 9. Cumulative return rate of HML.

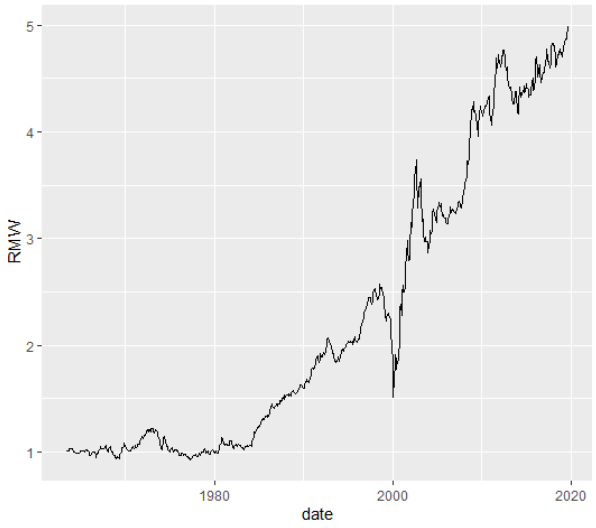

Figure 10. Cumulative return rate of RMW.

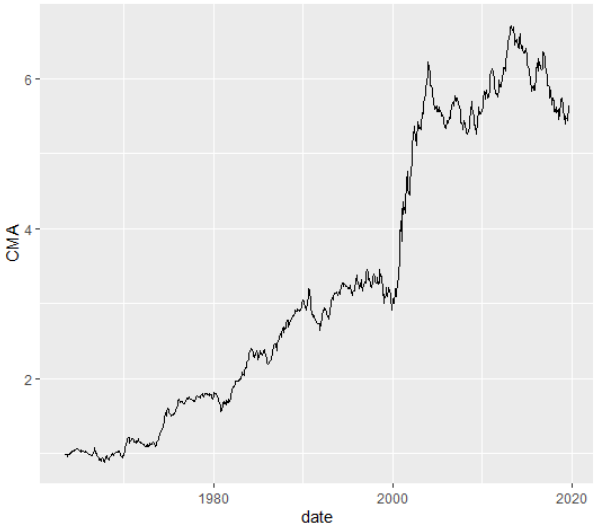

Figure 11. Cumulative return rate of CMA.

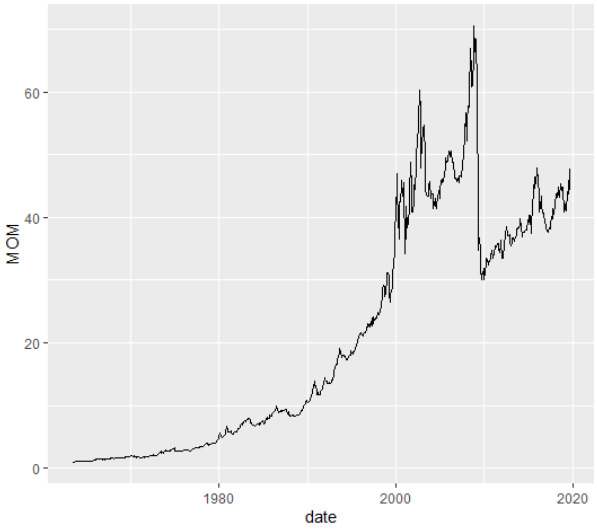

Figure 12. Cumulative return rate of MOM.

As can been seen from figure 7 to figure 12, the cumulative return rate of MKT reached 1748%, SMB had the lowest cumulative return rate of 241.4%, HML 519.8%, RMW 398.6%, CMA 463.4%, and MOM had the highest cumulative return rate which is 4335%.

Following the formula of SMB and MOM, we can find that the income of small stock is slightly higher than which of big stock, which means the size effect is not very significant. The difference between the futures returns of stocks with higher past returns and those with lower returns is large.

In the subsequent regression model analysis, we used the F test to judge whether the parameters in the model we expected were suitable for estimating the matrix. Followed is the process:

For the F- test, we make a hypothesis that is

$H_0: \beta_i = \gamma_i = \delta_i = \mu_i = \rho_i = \sigma_i = 0$

$H_1: \beta_i \neq 0$ or $\gamma_i \neq 0$ or $\delta_i \neq 0$ or $\mu_i \neq 0$ or $\rho_i \neq 0$ or $\sigma_i \neq 0$

By using R software, it can be obtained that the F critical value of 99% confidence level is 2.828989, and all 6 F statistics are far greater than 2.828989, so we reject the null hypothesis.

3. Method

To study the effects of candidate factors on the returns of different portfolios, our group introduced an index model and factor model and used R software to do the regression. In these two models, there are six candidate factors, which are MKT, SMB, HML, RMW, CMA, and MOM. MKT is the excess return on the value-weight market portfolio and SMB represents the excess return on a diversified portfolio of small stocks minus the excess return on a diversified portfolio of big stocks (i.e., the size effect). Besides, the excess return spread of cheap minus expensive stocks (i.e., the value effect) is called HML. RMW means the difference between the excess returns on diversified portfolios of stocks with robust and weak profitability while the excess return spread of firms that invest conservatively minus aggressively is denoted by CMA. In addition, MOM signifies the excess return spread of firms with high prior return minus low prior return.

We first used the index model which includes all six factors above and compared the coefficients of these six factors to find out which factor is more influential to the portfolio returns. It can be expressed as follows:

$$R_{it} - R_{ft} = \alpha_i + \beta_i MKT_t + \gamma_i SMB_t + \delta_i HML_t + \mu_i RMW_t + \rho_i CMA_t + \sigma_i MOM_t + \epsilon_{it} \quad (1)$$

Where R_{it} is the return on portfolio i for period t and R_{ft} stands for the risk-free rate. If the sensitivities to the five factorsβ_i γ_i δ_i $\mu_i$and$\rho_i$capture all variation in expected returns, the intercept α_i is zero for all portfolios i.

Then, by observing the results of the regression of the index model, we found that the coefficients of MOM, CMA, and RMW is small compared with other factors. Considering the different levels of impacts of factors on our model, to further study the influences of factors, we neglected MOM, CMA, and RMW and applied the factor models with three (MKT, SMB, HML) and five factors (MKT, SMB, HML, RMW, CMA) respectively. After calculation, the value of adjusted R^2 was compared with that in the index model. Models are shown as follows:

Three factor model:

$$R_{it} - R_{ft} = \alpha_i + \beta_i MKT_t + \gamma_i SMB_t + \delta_i HML_t + \epsilon_{it} \quad (2)$$

Five factor model:

$$R_{it} - R_{ft} = \alpha_i + \beta_i MKT_t + \gamma_i SMB_t + \delta_i HML_t + \mu_i RMW_t + \rho_i CMA_t + \epsilon_{it} \quad (3)$$

4. Results analysis

Table 2. Six-factor model Index Model.

	Excess Returns					
	Port1	Port2	Port3	Port4	Port5	Port6
	(1)	(2)	(3)	(4)	(5)	(6)
MKT	1.056*** (1.043, 1.069)	0.972*** (0.962, 0.983)	1.000*** (0.991, 1.010)	0.996*** (0.986, 1.006)	1.006*** (0.987, 1.025)	1.051*** (1.034, 1.068)
SMB	0.995*** (0.977, 1.012)	0.843*** (0.829, 0.858)	0.879*** (0.866, 0.892)	-0.107*** (-0.121, -0.093)	-0.104*** (-0.131, -0.078)	0.008 (-0.015, 0.031)
HML	-0.371*** (-0.397, -0.346)	0.202*** (0.181, 0.223)	0.533*** (0.514, 0.552)	-0.270*** (-0.290, -0.250)	0.245*** (0.208, 0.283)	0.826*** (0.793, 0.859)
RMW	-0.161*** (-0.186, -0.136)	0.099*** (0.079, 0.120)	0.061*** (0.042, 0.080)	0.160*** (0.140, 0.180)	0.136*** (0.099, 0.173)	-0.062*** (-0.094, -0.029)
CMA	-0.085*** (-0.122, -0.048)	0.081*** (0.050, 0.111)	0.075*** (0.047, 0.102)	-0.022 (-0.051, 0.007)	0.208*** (0.153, 0.262)	-0.182*** (-0.230, -0.134)
MOM	-0.028*** (-0.040, -0.015)	-0.016** (-0.026, -0.005)	-0.003 (-0.012, 0.006)	-0.005 (-0.015, 0.005)	-0.026** (-0.044, -0.007)	-0.030*** (-0.046, -0.014)
Constant	-0.072** (-0.124, -0.020)	0.047* (0.004, 0.090)	0.043* (0.004, 0.082)	0.077*** (0.036, 0.119)	-0.124*** (-0.201, -0.047)	-0.037 (-0.105, 0.031)
Observations	675	675	675	675	675	675
R^2	0.987	0.986	0.989	0.982	0.928	0.957
Adjusted R^2	0.987	0.986	0.989	0.982	0.927	0.956
Residual Std. Error (df = 668)	0.778	0.641	0.579	0.612	1.145	1.005
F Statistic (df = 6; 668)	8,328.943***	7,796.261***	10,205.470***	6,094.682***	1,437.360***	2,468.817***
Note:			*p<0.1; **p<0.05; ***p<0.01			

We used software R to obtain the six-factor model, five-factor model, and three-factor model respectively, as shown in Table 2, Table 3and Table 4.

In the regression of the six-factor model, except for the insignificant SMB coefficient of portfolio 6 and the insignificant CMA coefficient of portfolio 4, the MKT, SMB, HML, RMW, and CMA coefficients of the six portfolios are all significant at 99% confidence level. For the factor MOM, only the MOM coefficients of port1 and port6 were significant at 99% significance level, those of port2 and port5 were significant at 95% significance

Table 3. Five-factor Model.

	Excess Returns					
	Port1 (1)	Port2 (2)	Port3 (3)	Port4 (4)	Port5 (5)	Port6 (6)
MKT	1.060***	0.975***	1.001***	0.997***	1.010***	1.055***
	(1.047, 1.073)	(0.964, 0.985)	(0.991, 1.010)	(0.986, 1.007)	(0.991, 1.029)	(1.039, 1.072)
SMB	0.993***	0.843***	0.879***	-0.108***	-0.106***	0.006
	(0.975, 1.011)	(0.828, 0.857)	(0.866, 0.892)	(-0.122, -0.094)	(-0.132, -0.079)	(-0.017, 0.030)
HML	-0.356***	0.211***	0.534***	-0.267***	0.259***	0.842***
	(-0.381, -0.331)	(0.190, 0.231)	(0.516, 0.553)	(-0.286, -0.248)	(0.222, 0.295)	(0.810, 0.874)
RMW	-0.167***	0.096***	0.060***	0.159***	0.130***	-0.069***
	(-0.193, -0.142)	(0.075, 0.117)	(0.041, 0.079)	(0.139, 0.179)	(0.093, 0.167)	(-0.101, -0.036)
CMA	-0.095***	0.075***	0.074***	-0.024	0.198***	-0.193***
	(-0.132, -0.058)	(0.045, 0.106)	(0.046, 0.101)	(-0.053, 0.005)	(0.144, 0.253)	(-0.241, -0.145)
Constant	-0.092***	0.036	0.041*	0.074***	-0.142***	-0.059
	(-0.144, -0.040)	(-0.007, 0.078)	(0.002, 0.079)	(0.033, 0.114)	(-0.219, -0.066)	(-0.126, 0.009)
Observations	675	675	675	675	675	675
R^2	0.987	0.986	0.989	0.982	0.928	0.956
Adjusted R^2	0.986	0.986	0.989	0.982	0. 927	0.956
Residual Std. Error (df = 669)	0.785	0.644	0.579	0.612	1.148	1.012
F Statistic (df = 5; 669)	9,807.901***	9,281.507***	12,260.230***	7,316.178***	1,712.742***	2,923.333***
Note:			*p<0.1; **p<0.05; ***p<0.01			

Level and those of port3 and port4 were not. As for the significance of the constants, the constants of port4 and port5 are significant at 99% confidence level, the constants of port1 are significant at 95% confidence level, the constants of port2 and port3 are significant at 90% confidence level, and the constants of port6 are insignificant.

For the five-factor model regression, as in the six-factor regression model, except for the insignificant SMB coefficient of portfolio 6 and the insignificant CMA coefficient of portfolio 4, the MKT, SMB, HML, RMW and CMA coefficients of the six portfolios are all significant at 99% confidence level. In terms of the significance of the constants, port1, port4 and port5 are significant at 99% confidence level, port3 is significant at 90% confidence level, and port2 and port6 are insignificant.

Table 4. Three-factor Model.

	Excess Returns					
	Port1	Port2	Port3	Port4	Port5	Port6
	(1)	(2)	(3)	(4)	(5)	(6)
MKT	1.081***	0.960***	0.989***	0.988***	0.981***	1.080***
	(1.068, 1.094)	(0.950, 0.971)	(0.980, 0.998)	(0.977, 0.999)	(0.962, 0.999)	(1.064, 1.096)
SMB	1.030***	0.821***	0.866***	-0.143***	-0.135***	0.022
	(1.011, 1.049)	(0.807, 0.836)	(0.853, 0.879)	(-0.158, -0.128)	(-0.161, -0.108)	(-0.001, 0.045)
HML	-0.400***	0.245***	0.568***	-0.277***	0.349***	0.755***
	(-0.420, -0.381)	(0.230, 0.261)	(0.554, 0.582)	(-0.293, -0.261)	(0.321, 0.377)	(0.730, 0.779)
Constant	-0.168***	0.083***	0.075***	0.124***	-0.060	-0.119***
	(-0.223, -0.113)	(0.040, 0.127)	(0.037, 0.114)	(0.080, 0.169)	(-0.137, 0.017)	(-0.187, -0.052)
Observations	675	675	675	675	675	675
R^2	0.984	0.984	0.989	0.977	0.921	0.953
Adjusted R^2	0.984	0.984	0.989	0.977	0.921	0.953
Residual Std. Error (df = 671)	0.853	0.673	0.595	0.693	1.195	1.046
F Statistic (df = 3; 671)	13,820.250***	14,133.500***	19,329.500***	9,460.679***	2,617.023***	4,538.351***
Note:			*p<0.1; **p<0.05; ***p<0.01			

In the regression of the three-factor model, the coefficients of three factors in the six combinations are significant at 99% confidence level, except for the SMB coefficient of combination 6, which is not significant. In addition, all the constants in the six portfolios are significant at the 99% confidence level, except for the constants in portfolio 5 that are not significant.

In the regression of the six-factor model, the coefficients of MKT, SMB and HML were significantly larger than those of RMW, CMA, and MOM, especially the coefficients of MKT and SMB. Moreover, the coefficients of MKT, SMB, and HML of the six portfolios were significant at 99% confidence level, except for the SMB coefficient of portfolio 6. Using R software, the correlation between 6 factors can be obtained, as shown in Table 5.

Table 5. Correlation of different factor.

	MKT	SMB	HML	RMW	CMA	MOM
MKT	1	0.277	-0.250	-0.227	-0.385	-0.140
SMB	0.277	1	-0.064	-0.347	-0.104	-0.035
HML	-0.250	-0.064	1	0.063	0.694	-0.197
RMW	-0.227	-0.347	0.063	1	-0.035	0.109
CMA	-0.385	-0.104	0.694	-0.035	1	-0.028
MOM	-0.140	-0.035	-0.197	0.109	-0.028	1

Table 6. The Vif (variance inflation factor) of factor.

MKT	SMB	HML	RMW	CMA	MOM
1.332	1.200	2.112	1.225	2.244	1.109

It can be found that the absolute values of most of the correlation coefficients are less than 0.30, indicating that there is only a very weak positive or negative correlation between them. Only RMW

and SMB, CMA and MKT, and CMA and HML have correlation coefficients greater than 0.30. The correlation coefficient between RMW and SMB is -0.347, which is very close to -0.30, so there is still a weak negative correlation between RMW and SMB. The correlation coefficient between CMA and MKT is -0.385, which is also close to -0.30, so there is still a weak negative correlation between CMA and MKT. Furthermore, the correlation coefficient between CMA and HML is 0.694, which is close to 0.70, so there is a strong positive correlation between CMA and HML. VIF (variance inflation factor) can also be get by using software R, as shown in Table 6. And all 6 VIFs were between 1 and 5, indicating a moderate correlation.

F-test also be used, the hypothesis is $H_0: \beta_i = \gamma_i = \delta_i = \mu_i = \rho_i = \sigma_i = 0$; $H_1: \beta_i \neq 0$ or $\gamma_i \neq 0$ or $\delta_i \neq 0$ or $\mu_i \neq 0$ or $\rho_i \neq 0$ or $\sigma_i \neq 0$. Using R software, it can be obtained that the F critical value of 99% confidence level is 2.828989, and all 6 F statistics are far greater than 2.828989, so the null hypothesis needs to be rejected. Also, the R^2 of all six portfolios is greater than 90%, which means that more than 90% of the data fit the regression model. The estimation results agree well with the model. The adjusted R^2 analyses whether additional input variables contribute to the model. Therefore, the adjusted R^2 in the six-factor model can be compared with the adjusted R^2 in the three-factor model and the five-factor model. The adjusted R^2 in the three-factor model and the five-factor model is very large and very close to the corresponding adjusted R^2 in the six-factor model. This indicates that the additional factors RMW, CMA, and MOM contribute little to our model.

5. Conclusion

The factor model is crucial to the financial market. Given the data derived from Yahoo Finance that have monthly prices for six portfolios and six candidate factors from the time span of 1963 to 2019, we want to explore the relevance between our candidate factors and the portfolio return and determine the candidate factors that have the most effects on the return. By tackling such a topic, the group employs the Three-Factor Model and Five-Factor Model and runs the regression analysis in R to observe the relevance.

In descriptive statistical analysis, R software was used to draw the descriptive statistics table and the discount charts of monthly return rate and cumulative return rate of 6 factors and the formulas of factors were combined to analyze the table and image. In the regression part, R software was also used to establish the regression model and obtain various parameters of the model, such as R^2 and adjustmentR^2. In the following analysis, R software was used to conduct F test, calculate variance inflation factor and conduct analysis according to the obtained data.

In conclusion, our results show that except for several cases, most of the MKT, SMB, HML, RMW, and CMA coefficients are significant at 99% intervals while the coefficients of factor MOM are only significant in several specific portfolios. Moreover, among all six candidate factors, MKT has the strongest positive effects on the return of each portfolio and MOM influences the investment returns negatively but slightly. It shows that SMB and HML are secondary relevant to the portfolio returns and they can either improve or discourage the returns depend on disparate portfolios. Besides, RMW and SMA also make a difference positively or negatively on different combinations, but the impacts seemed to be small according to the results of our models.

The biggest disadvantage of using R^2 to select the model is that it has no theoretical basis in likelihood (which muddles the factor ratio) and only the R2 model is valid when it is defined (e.g., linear and non-generalized linear models). AIC or BIC are better solutions because they are based on information/ probability theory and apply to all generalized linear models. However, the three-factor model is still unable to adequately explain time-series fluctuations in portfolio returns, indicating that there is still potential for improvement.

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